

Inference at * 1 0
of proof for Lemma minus_imin:

1. $a : \mathbb{Z}$
2. $b : \mathbb{Z}$
 $\vdash (-\text{if } a \leq_z b \text{ then } a \text{ else } b \text{ fi}) = \text{if } (-a) \leq_z -b \text{ then } -b \text{ else } -a \text{ fi}$
by PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
12:n,
13:n,
14:n,
15:n,
16:n,
17:n,
18:n,
19:n,
20:n,
18:n,
21:n,
19:n,
22:n,
23:n,
24:n,
25:n,
26:n,
24:n,
23:n,
27:n,
28:n,
29:n,
30:n,
28:n,
31:n,
29:n,
32:n,

33:n,
 34:n,
 35:n,
 36:n,
 34:n,
 33:n,
 37:n,
 38:n,
 39:n,
 40:n,
 41:n,
 42:n,
 43:n,
 44:n,
 45:n,
 46:n,
 47:n,
 48:n,
 46:n,
 49:n,
 47:n,
 50:n,
 51:n,
 52:n,
 53:n,
 54:n,
 52:n,
 51:n,
 55:n}

1:wf..... NILNIL

$\vdash a \leq_z b \in \mathbb{B}$

2:wf..... NILNIL

$\vdash \mathbb{B} \in \text{Type}$

3:wf..... NILNIL

3. $a \leq_z b = \text{tt}$

$\vdash (a \leq_z b = \text{tt}) \in \mathbb{P}_1$

4:wf..... NILNIL

3. $a \leq_z b = \text{tt}$

$\vdash (\uparrow a \leq_z b) \in \mathbb{P}_1$

5:wf..... NILNIL

3. $a \leq_Z b = \text{tt}$
 $\vdash (a \leq b) \in \mathbb{P}_1$
6:wf..... NILNIL

3. $a \leq_Z b = \text{tt}$
 $\vdash a \leq_Z b \in \mathbb{B}$
7:wf..... NILNIL

3. $a \leq_Z b = \text{tt}$
 $\vdash a \in \mathbb{Z}$
8:wf..... NILNIL

3. $a \leq_Z b = \text{tt}$
 $\vdash b \in \mathbb{Z}$
9:wf..... NILNIL

3. $a \leq b$
 $\vdash (-a) \leq_Z -b \in \mathbb{B}$
10:wf..... NILNIL

3. $a \leq b$
 $\vdash \mathbb{B} \in \text{Type}$
11:wf..... NILNIL

3. $a \leq b$
4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash ((-a) \leq_Z -b = \text{tt}) \in \mathbb{P}_1$
12:wf..... NILNIL

3. $a \leq b$
4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (\uparrow(-a) \leq_Z -b) \in \mathbb{P}_1$
13:wf..... NILNIL

3. $a \leq b$
4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash ((-a) \leq (-b)) \in \mathbb{P}_1$
14:wf..... NILNIL

3. $a \leq b$
4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (-a) \leq_Z -b \in \mathbb{B}$
15:wf..... NILNIL

3. $a \leq b$
4. $(-a) \leq_Z -b = \text{tt}$

$\vdash (-a) \in \mathbb{Z}$
 16:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (-b) \in \mathbb{Z}$
 17:truecase..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq (-b)$
 $\vdash (-a) = (-b)$
 18:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash ((-a) \leq_Z -b = \text{ff}) \in \mathbb{P}_1$
 19:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (\uparrow(-b) <_Z (-a)) \in \mathbb{P}_1$
 20:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash ((-b) < (-a)) \in \mathbb{P}_1$
 21:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (\uparrow(\neg_b(-a) \leq_Z -b)) \in \mathbb{P}_1$
 22:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (-a) \leq_Z -b \in \mathbb{B}$
 23:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (-a) \in \mathbb{Z}$
 24:wf..... NILNIL

 3. $a \leq b$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (-b) \in \mathbb{Z}$

25:antecedent..... NILNIL

- 3. $a \leq b$
- 4. $(-a) \leq_z -b = \text{ff}$
- $\vdash \text{True}$

26:wf..... NILNIL

- 3. $a \leq b$
- 4. $(-a) \leq_z -b = \text{ff}$
- 5. $(\uparrow(\neg_b(-a) \leq_z -b)) = (\uparrow(-b) <_z (-a))$
- $\vdash \mathbb{P}_1 = \mathbb{P}_1$

27:falsecase..... NILNIL

- 3. $a \leq b$
- 4. $(-b) < (-a)$
- $\vdash (-a) = (-a)$

28:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash (a \leq_z b = \text{ff}) \in \mathbb{P}_1$

29:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash (\uparrow b <_z a) \in \mathbb{P}_1$

30:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash (b < a) \in \mathbb{P}_1$

31:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash (\uparrow(\neg_b a \leq_z b)) \in \mathbb{P}_1$

32:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash a \leq_z b \in \mathbb{B}$

33:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash a \in \mathbb{Z}$

34:wf..... NILNIL

- 3. $a \leq_z b = \text{ff}$
- $\vdash b \in \mathbb{Z}$

35:antecedent..... NILNIL

3. $a \leq_Z b = \text{ff}$
 $\vdash \text{True}$
 36:wf..... NILNIL

3. $a \leq_Z b = \text{ff}$
 4. $(\uparrow(\neg_b a \leq_Z b)) = (\uparrow b <_Z a)$
 $\vdash \mathbb{P}_1 = \mathbb{P}_1$
 37:wf..... NILNIL

3. $b < a$
 $\vdash (-a) \leq_Z -b \in \mathbb{B}$
 38:wf..... NILNIL

3. $b < a$
 $\vdash \mathbb{B} \in \text{Type}$
 39:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash ((-a) \leq_Z -b = \text{tt}) \in \mathbb{P}_1$
 40:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (\uparrow(-a) \leq_Z -b) \in \mathbb{P}_1$
 41:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash ((-a) \leq (-b)) \in \mathbb{P}_1$
 42:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (-a) \leq_Z -b \in \mathbb{B}$
 43:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (-a) \in \mathbb{Z}$
 44:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{tt}$
 $\vdash (-b) \in \mathbb{Z}$
 45:truecase..... NILNIL

3. $b < a$
 4. $(-a) \leq (-b)$
 $\vdash (-b) = (-b)$
 46:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash ((-a) \leq_Z -b = \text{ff}) \in \mathbb{P}_1$
 47:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (\uparrow(-b) <_Z (-a)) \in \mathbb{P}_1$
 48:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash ((-b) < (-a)) \in \mathbb{P}_1$
 49:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (\uparrow(\neg_b(-a) \leq_Z -b)) \in \mathbb{P}_1$
 50:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (-a) \leq_Z -b \in \mathbb{B}$
 51:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (-a) \in \mathbb{Z}$
 52:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash (-b) \in \mathbb{Z}$
 53:antecedent..... NILNIL

3. $b < a$
 4. $(-a) \leq_Z -b = \text{ff}$
 $\vdash \text{True}$
 54:wf..... NILNIL

3. $b < a$
 4. $(-a) \leq_z -b = \text{ff}$
 5. $(\uparrow(\neg_b(-a) \leq_z -b)) = (\uparrow(-b) <_z (-a))$
 $\vdash \mathbb{P}_1 = \mathbb{P}_1$
 55:falsecase..... NILNIL

3. $b < a$
 4. $(-b) < (-a)$
 $\vdash (-b) = (-a)$
 .